Application of Global Sensitivity Equations in Multidisciplinary Aircraft Synthesis

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The present paper investigates the applicability of the Global Sensitivity Equation (GSE) method in the multidisciplinary synthesis of aeronautical vehicles. The GSE method provides an efficient approach for representing a large coupled system by smaller subsystems and accounts for the subsystem interactions by means of first-order behavior sensitivities. This approach was applied in an aircraft synthesis problem with performance constraints stemming from the disciplines of structures, aerodynamics, and flight mechanics. Approximation methods were considered in an attempt to reduce problem dimensionality and to improve the efficiency of the optimization process. The influence of efficient constraint representations, the choice of design variables, and design variable scaling on the conditioning of the system matrix was also investigated.

Nomenclature

A = system dynamics matrix
a = stringer cross-sectional areas
B = control matrix
b = wing semispan
C = wing chord length
d = static wingtip displacements
dd = dynamic displacements
G = gain matrix
H = gust distribution matrix
I_{yy} = moment of inertia
K = stiffness matrix
$\bar{K}$ = generalized stiffness matrix
L = lift vector
M = mass matrix
Q = generalized aerodynamic force matrix
q_∞ = dynamic pressure
S = wing area
Th = membrane thickness
u = control input vector
V = wing-box volume
W = weight
w_G = gust input vector
X = design variable
x = state variable
Y = sybsystem output
y = system output
δ = control surface deflection
γ = dihedral
Ω = cumulative constraint measure
ϕ = eigenvector
$\bar{\rho}$ = cumulative constraint constant
σ = static stress
ω = eigenvalue

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Introduction

A PRINCIPAL concern in the application of optimization methods in large engineering systems resides in examining approaches that allow for an efficient and accurate mathematical representation of such systems. This is especially pertinent in light of several recent publications[1,2] that indicate an increased interest in the problem of optimization in a multidisciplinary environment. Such an environment generally results in a system composed of distinct but intrinsically coupled disciplines that interact with one another in such a manner that perturbations in parameters pertinent to one discipline affect the output of others. This interdependence of subsystems contributes to difficulties in successfully implementing a holistic design synthesis strategy. Furthermore, such an integrated implementation is also subject to complexities introduced as a result of an increased number of design variables and constraints. A major objective of recent investigations has been to overcome the many obstacles inherent in the multidisciplinary design problem in order to take advantage of the synergistic nature of integrated design.

Two basic solution strategies have been proposed for integrated, multidisciplinary design problems. The first involves an ad hoc decomposition in which the participating analyses of the various subsystems are performed in some prescribed order. In such an approach, the resulting design is dependent on the order in which analyses are carried out. The more desirable strategy is one which embraces parallel processing, in which each subsystem is examined simultaneously and with due consideration of all subsystem interactions.

The recently proposed Global Sensitivity Equation (GSE) method[3,4] introduces an efficient approach for decoupling a large system into smaller subsystems in order to obtain first-order sensitivity of the behavior response. This sensitivity information may then be used to construct linear approximations to the response for use in a parallel optimization environment. The GSE is particularly compatible with multidisciplinary design, as each discipline involved may be relegated to a subsystem, thus allowing for the decoupling of the problem.

The purpose of this investigation was to examine the multidisciplinary synthesis of an aeronautical vehicle in the context

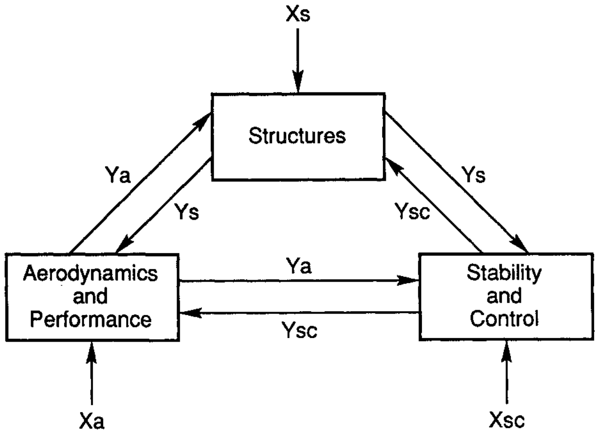

Fig. 1 Flowchart for subsystem interactions.

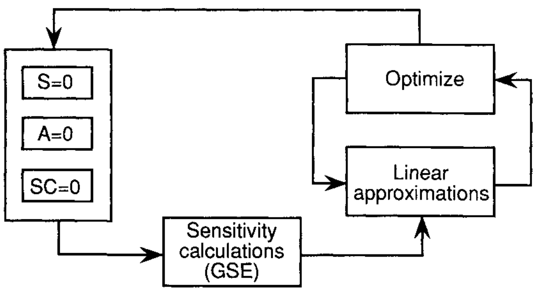

Fig. 2 Flowchart for multidisciplinary design synthesis.

of the GSE approach. The synthesis problem involved sizing of the vehicle for design constraints stemming from the requirements of structural integrity and response, aerodynamic performance, and prescribed stability and control requirements. As explained in subsequent sections of this paper, the multidisciplinary synthesis problem described above results in a large number of linear algebraic equations that must be solved to obtain the behavior sensitivity. In the present work, iterative solution techniques were examined for this application as an alternative to traditional matrix decomposition approaches. Approximation concepts including efficient methods of constraint treatment[5] were investigated to reduce problem dimensionality and to improve the efficiency of the optimization process. The influence of choice of design variables and design variable scaling and of use of constraint reduction techniques on the conditioning of the system matrix was also investigated.

System Decomposition by the Global Sensitivity Equation Method

The underlying concepts in the proposed approach for decomposition are simple and make use of the fact that the first derivative of a nonlinear function at a point is equal to the first derivative of the function linear approximation at that point. For the case where there is interaction between the disciplines of structures, aerodynamics, and stability/control as shown in Fig. 1, the analysis equations can be expressed in a symbolic form as follows:

$$Y_S = Y_S(X_S, Y_A, Y_{SC})$$

$$Y_A = Y_A(X_A, Y_S, Y_{SC})$$

$$Y_{SC} = Y_{SC}(X_{SC}, Y_A, Y_S) \tag{1}$$

Here, X_S, X_A, and X_{SC} are the variables local to the structures, aerodynamics, and stability/control systems, respectively. Y_A is the output vector for the aerodynamic system and, in the most general form of coupling, this vector acts as a set of auxiliary input variables for the structures and stability/control systems. Similarly, Y_S and Y_{SC} act as auxiliary input variables for the other systems. Thus, the vectors Y provide the coupling links between the three systems. It is possible to develop expressions[3] for the sensitivity of the output of each discipline to intrinsic variables X in the following matrix form:

$$\overset{\begin{matrix}\text{NS} & \qquad\text{NA} & \qquad\text{NSC}\end{matrix}}{\left[\begin{array}{c|c|c} I & -\dfrac{\partial Y_S}{\partial Y_A} & -\dfrac{\partial Y_S}{\partial Y_{SC}} \\ \hline -\dfrac{\partial Y_A}{\partial Y_S} & I & -\dfrac{\partial Y_A}{\partial Y_{SC}} \\ \hline -\dfrac{\partial Y_{SC}}{\partial Y_S} & -\dfrac{\partial Y_{SC}}{\partial Y_S} & I \end{array}\right]} \left\{\begin{array}{c} \dfrac{dY_S}{dX_S} \\ \hline \dfrac{dY_A}{dX_S} \\ \hline \dfrac{dY_{SC}}{dX_S} \end{array}\right\} = \left\{\begin{array}{c} \dfrac{\partial Y_S}{\partial X_S} \\ \hline 0 \\ \hline 0 \end{array}\right\} \tag{2}$$

where NS, NA, and NSC represent the dimensionality of the output vectors from the structures, aerodynamic, and stability/control disciplines, respectively. Similar sensitivities can be developed for changes in the intrinsic variables of the aerodynamics and stability and control disciplines.

Note that the total derivatives dY_S/dX_S, dY_A/dX_S, and dY_{SC}/dX_S can be solved from the above set of equations if the partial derivatives that appear in the coefficient matrix and in

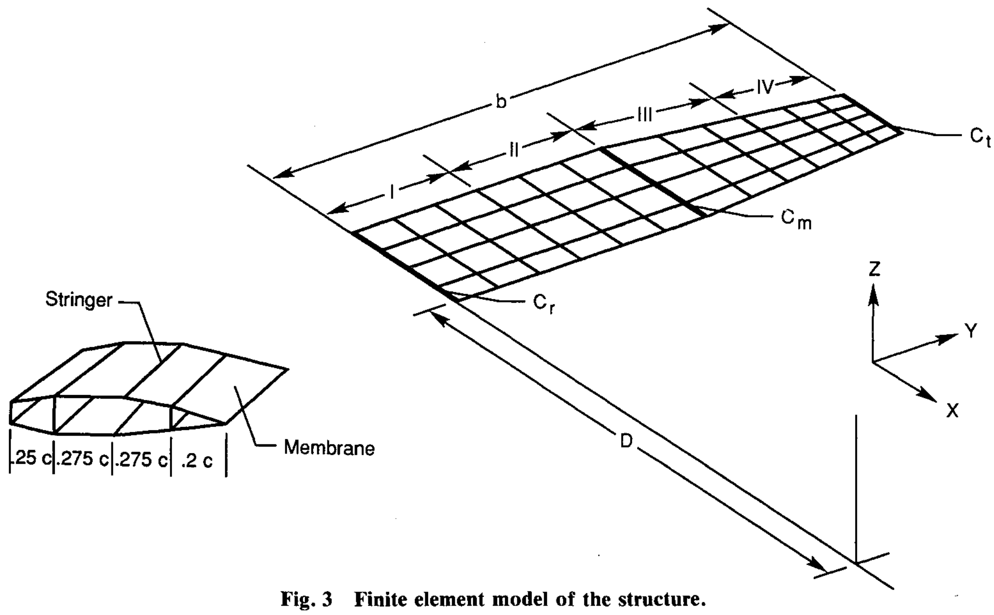

Fig. 3 Finite element model of the structure.

the right-hand vector are known. These partial sensitivities can be computed locally within the system, eliminating the need to perform computationally expensive interdisciplinary iteration. This also diminishes the possibility of errors associated with roundoff and truncation in the iterative process from having adverse effects on the quality of the sensitivity results. It is worthwhile to note that the output from the analysis of one discipline may contain data that has no influence on other disciplines. As an example, the output from the structures analysis may include modal information that is passed as input to both the aerodynamics and flight mechanics disciplines. It may also include data such as the objective and constraint function information that is not passed as input, as it has no influence on the analyses of those disciplines.

The sensitivities obtained from the preceding analysis can be used to develop linear approximations to the output response of each subsystem, which can be subsequently employed in an optimization process. However, due to the complexity of the multidisciplinary problem, the dimensionality of the local sensitivity matrices may be prohibitively large for repetitive decomposition in an optimization sequence and may contribute to substantial reductions in the numerical efficiency.

Multidisciplinary Synthesis Problem

The objective of the design problem was to size the structure and planform geometry of a general aviation aircraft for minimum weight, subject to design constraints stemming from requirements of structural integrity, aerodynamic performance, and flight mechanics performance characteristics. The design synthesis methodology for the integrated problem is shown in Fig. 2. Since there exists an inherent coupling between the three disciplines, an initial design point must be first determined iteratively. Based on this starting point, total behavior sensitivities were determined by application of the GSE method. These sensitivities were used to construct piecewise linear approximations for the behavior response for use in a feasible-usable search direction optimization algorithm. Such an optimization was performed with prescribed move limits on the design variables, necessary to ensure the validity of the linear approximation. The process was repeated until specified convergence criteria were met.

Model Definition and Analysis Methodology

The multidisciplinary environment of structures, aerodynamics, and flight mechanics was represented in terms of distinct analysis models. The scope of analytical detail and the primary tools used to represent that detail are described in this section. Description of the design problem in terms of the subsystem design variables and constraints is also included in this section.

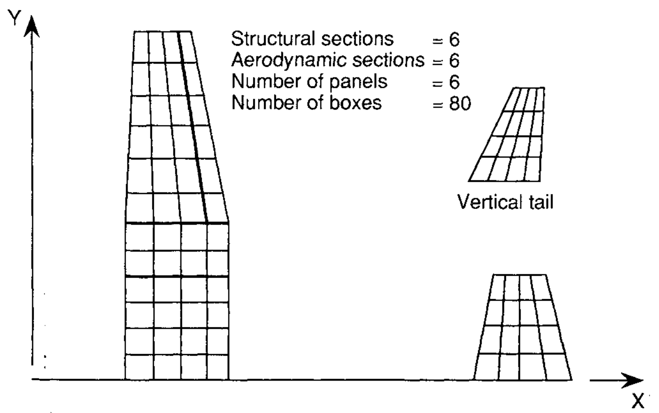

Fig. 4 Paneling for the aerodynamic model.

Structures Subsystem

The finite element analysis model for the structures discipline is shown in Fig. 3. A stick model of the fuselage and tail structure represented by beam elements was connected to a built-up membrane/stringer model for the wing structure. A symmetric half of the structural model was used with a total of 426 degrees of freedom. Basic configuration and geometry design variables included the wingspan, wing dihedral, and location of the wing along the fuselage. The structural sizing variables included the stringer cross-sectional areas, membrane thicknesses, and chord lengths at three prescribed spanwise locations. Design constraints were placed on the first and second natural frequencies of the structure, wingtip displacements, membrane and stringer stresses at the root section, and internal volume of the wing structure.

All structural analysis pertinent to the problem was performed using the finite element program Engineering Analysis Language (EAL).[6] EAL is a high-order language with primary applications in analysis and design of solid and fluid systems based on a finite element representation of the analysis domain. Individual processors communicate through a data base containing information describing the finite element model of the structure as well as data accumulated during execution of the runstream.

Aerodynamics and Performance Subsystem

Definition of the aerodynamic model was in accordance with the input requirements of an unsteady doublet lattice program Interactions between Flexible Structures, Unsteady Aerodynamic and Active Controls (ISAC)[7] and is shown in Fig. 4. A beam representation for the fuselage was retained. A lifting surface, defined as the aggregate of the upper and lower surfaces of a wing with a NACA 2412 airfoil, was modeled with plate elements, as was the tail assembly. The aerodynamic design variables included the span length, location of wing along the fuselage, wing dihedral, and chord lengths at selected spanwise stations. Four design performance constraints were introduced in this subsystem. These included bounds on the stall velocity, landing and takeoff distances, and the range of the aircraft.

To study the forced motion of the aircraft in the frequency domain, the generalized aerodynamic forces were obtained as follows

$$[Q(k,Ma)] = [\phi]^T [\Delta P(k,Ma)] \tag{3}$$

where k is the reduced frequency, $[\phi]$ is the modal matrix, Ma is the Mach number, and ΔP is the differential pressure over the surface generated due to motion and gust incidence. The gust time history was modeled as a deterministic sharp-edged gust.

Performance requirements were stated as constraints in the optimization problem and were determined from well-documented relations.[8] The stall velocity was found from the relation

$$V_S = \left[(2W)/(\rho C_{L_{\max}} S)\right]^{1/2} \tag{4}$$

where W is the weight of the aircraft at takeoff, $C_{L_{\max}}$ is the maximum lift coefficient, ρ is the density of air at sea level, and S is the wing area. Another performance requirement was the landing distance over a 50-ft obstacle and was calculated by summing the distance in the air D_A and the distance on the ground D_G, defined as follows:

$$D_A = (W/F)/\left[(V_{50}^2 - V_L^2)/2 + 50\right] \tag{5}$$

and

$$D_G = -V_L^2/(2a) \tag{6}$$

The quantity (W/F) is the average resistance coefficient, a is the uniform acceleration on the ground, V_L is the velocity at landing, and V_{50} is the velocity at the 50-ft height. Similarly, the takeoff distance over a 50-ft obstacle (D_{T0}) was found from

$$D_{T0} = (100K)/9 - 1000/3 \tag{7}$$

where

$$K = W^2/\left(S\, HP\, C_{L_{T0}}\Gamma\right) \tag{8}$$

and HP and Γ are the rated horsepower of the engine and the ratio of air densities, respectively, and $C_{L_{T0}}$ is the lift coefficient at takeoff.

Breguet's formula was used to determine the range. Based on the assumption that all fuel is used, the range R can be expressed as

$$R = 375(C_L/C_D)(N/F)\ln(W/W_p) \tag{9}$$

where N is propeller efficiency factor, F is the specific fuel consumption, and W_P is the difference between the takeoff weight and the weight of the fuel.

Stability/Control Subsystem

A first-order state space representation was used for the analytical model of the flight mechanics subsystem. The equation of motion for a structure with active controls and subject to time varying air loads can be written in terms of air loads Q_i and modal displacements q_i as follows:

$$M_i\ddot{q}_i(t) + \omega_i^2 M_i q_i(t) + \sum_{j=1}^{n} Q_{ij}q_i(t) = -Q_{i\delta}\delta(t) - q_i w_G(t) \tag{10}$$

where $\delta(t)$ is the control surface deflection and $w_G(t)$ is the gust velocity.

The dimensionality of the modal matrix is determined by the number of modes that are deemed necessary to model the structural displacements and other system degrees of freedom. Since the lower modes are dominant in representing the displacements, typically only the first six to ten modes are used in the analysis. In the present work, one, three, and five modes were used for this purpose.

A first-order state space representation of the governing differential equations for the open-loop system can be written as follows:

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} + [H]\{n\} \tag{11}$$

where A is the system dynamics matrix, B and H are the control and gust distribution matrices, respectively, x is the state variable vector, u is the control input vector, and n is the gust vector.

In terms of the state vector x, the system output y can be written as

$$\{y\} = [C]\{x\} \tag{12}$$

where the matrix $[C]$ contains information specific to the location of the sensors. The output vector is of length s, where s is the number of sensors present in the system.

The optimal state feedback control law can then be found as a function of the gain matrix as follows.

$$\{u\} = -[G]\{x\} \tag{13}$$

The use of this relationship in Eq. (25) yields an expression for determining the optimal state for the closed-loop system. A time-marching method was used to determine the time history for the state variables. Once the state solution is known, the dynamic displacements can be retrieved for each degree of freedom. The control input resulting from this analysis was used to determine the mass of the physical control system by referring to an empirical relationship between control power and control system mass. This mass was used as an input to the structural system and had a direct bearing on the dynamic characteristics through its influence on the structure modal behavior.

Control system analysis was performed through the use of a package of FORTRAN programs Optimal Regulator Algorithms for the Control of Linear Systems (ORACLS).[9] The package provides solutions to either discrete Linear Quadratic Gaussian (LQG) or time-invariant continuous problems.

The design variables for this subsystem included the span, chord lengths at designated spanwise locations, wing location along the fuselage, and wing dihedral. Note that the gain components in the controls analysis could be included as design variables and would directly influence the resulting control effort. Design constraints were imposed on control surface deflections, dynamic tip displacements, and settling times in a longitudinal oscillatory motion. The required stability derivatives for this analysis were obtained from a semielastic stability analysis, which involves a modification of rigid-body stability characteristics to account for structural deformations. The relationship

$$[\bar{K}]\{q\} + q_\infty[Q]\{q\} = 0 \tag{14}$$

between the generalized stiffness and generalized aerodynamic force matrices yields an elastic generalized aerodynamic force vector as a function of rigid and elastic terms such that

$$\begin{Bmatrix} \bar{Q}_{12} \\ \bar{Q}_{22} \end{Bmatrix} = \begin{Bmatrix} Q_{12} \\ Q_{22} \end{Bmatrix} - q_\infty \begin{bmatrix} Q_{13}\cdots Q_{1n} \\ Q_{23}\cdots Q_{2n} \end{bmatrix} \begin{bmatrix} \bar{K} + q_\infty Q \end{bmatrix}^{-1} \begin{Bmatrix} Q_{32} \\ \vdots \\ Q_{n2} \end{Bmatrix} \tag{15}$$

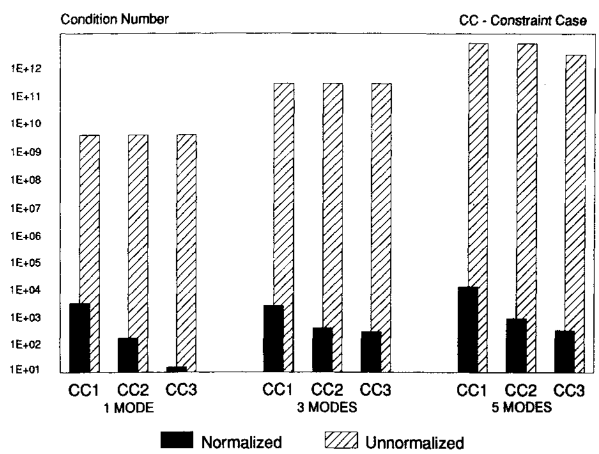

Fig. 5a Variation of condition number with system dimensionality and with constraint representation.

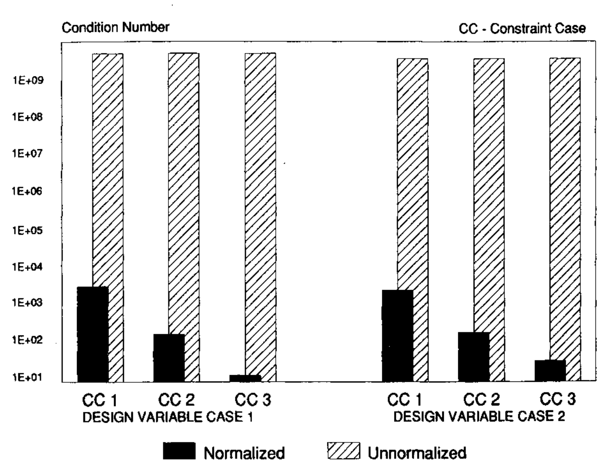

Fig. 5b Variation in condition number with design variable and constraint representation.

where n is the number of elastic and rigid-body modes, and the first two rows of $[Q]$ correspond to pitch and plunge modes.

Selected stability derivatives were determined in terms of the semielastic generalized aerodynamic force vector. These were used in the characteristic equation resulting from the stability analysis of longitudinal motion and from which the eigenvalues for this motion were determined. The time-to-half for longitudinal pitching motion was determined from the relation

$$t^{1/2} = 0.69t^*/\zeta \tag{16}$$

where ζ is the eigenvalue for the mode under consideration and

$$t^* = c/(2U) \tag{17}$$

defined in terms of the mean chord length c and the velocity U.

The rigid-body stability and control analysis was performed using a modified version of programs available in Ref. 9.

Implementation of Solution Techniques

In the use of the GSE method for the design problem defined above, the dimensionality of the global sensitivity matrix is of some concern. If each subsystem represents a discipline in a multidisciplinary optimization problem, it is conceivable that for a large number of outputs associated with each discipline, the dimensionality of the global sensitivity matrix can be potentially quite large. The system of linear algebraic equations that are obtained by recourse to the GSE method can be expressed as

$$[D]\{w\} = \{v\} \tag{18}$$

where $[D]$ is the GSE partitioned matrix containing the local sensitivity information of each subsystem, $\{v\}$ is a known column vector of partial sensitivities, and $\{w\}$ is the unknown column vector of total derivatives. If the vector $\{w\}$, required in forming the response approximations for each piecewise linear representation of the system, is obtained by decomposition of $[D]$, the process may become unacceptably expensive or inaccurate due to an accumulation of round-off errors. The present work adopted an alternative iterative solution to this system of equations. An initial approximation to the solution was assumed and successively modified to a converged solution. In this investigation, Gauss-Siedel iteration with relaxation was implemented to promote convergence.

Gauss-Siedel iteration is recursive in nature, as one repeatedly cycles through solutions for the unknowns which then replace the old values. Since the method uses the most recently calculated values for the unknowns, substantial computer storage savings may be achieved. A point relaxation technique was implemented, which essentially modified the calculated value for the unknowns before being stored. The recursive relation for this approach may be expressed as

$$w_i^{(\ell+1)} = w_i^{(\ell)} + \lambda\left[w_i^{(\ell+1)*} - w_i^{(\ell)}\right] \tag{19}$$

where λ is the relaxation factor, $(\ell+1)$ is the current iteration, and $w_i^{(\ell+1)*}$ is the value for the unknown obtained by Gauss-Siedel method in the current iteration.

The level of ill-conditioning associated with matrix $[D]$ may be expressed in terms of a condition number, which, in the present work, was defined as follows:

$$\|D\| \cdot \|D^{-1}\| \tag{20}$$

Here, the following definition for the first norm of $[D]$ was used:

$$\|D\|_1 = \max_j \sum_1 |d_{ij}| \tag{21}$$

The quantity $\|D^{-1}\|_1$ was estimated from the expression

$$\|D^{-1}\|_1 = \|z\|/\|y\|$$

where vector $\{y\}$ was chosen and $\{z\}$ was determined as follows:

$$[D]\{z\} = \{y\} \tag{22}$$

The condition number is a measure by which the accuracy of the solution may be gauged and is determined by the relative magnitude of the terms in the GSE matrix. Since the components of the output response vector Y and the design variable vector X are of varying magnitudes, it is important to scale the partial sensitivity terms in the GSE matrix. A normalization scheme was implemented that is most readily explained by considering two systems A and B, with scalar intrinsic design variables X_A and X_B and scalar output responses Y_A and Y_B. To determine the derivatives of Y_A and Y_B with respect to X_A, the global sensitivity equations can be written in the following matrix form:

$$\begin{bmatrix} I & -\dfrac{\partial Y_A}{\partial Y_B} \\ -\dfrac{\partial Y_B}{\partial Y_A} & I \end{bmatrix} \begin{Bmatrix} \dfrac{dY_A}{dX_A} \\ \dfrac{dY_B}{dX_A} \end{Bmatrix} = \begin{Bmatrix} \dfrac{dY_A}{dX_A} \\ 0 \end{Bmatrix} \tag{23}$$

The partial sensitivity terms appearing on the left and right sides of this equation were normalized as

$$\left(\frac{\partial Y_A}{\partial Y_B}\right)^* = \frac{Y_B}{Y_A}\frac{\partial Y_A}{\partial Y_B}$$
$$\left(\frac{\partial Y_B}{\partial Y_A}\right)^* = \frac{Y_A}{Y_B}\frac{\partial Y_B}{\partial Y_A}$$
$$\left(\frac{\partial Y_A}{\partial X_A}\right)^* = \frac{X_A}{Y_A}\frac{\partial Y_A}{\partial X_A} \tag{24}$$

and the normalized global sensitivity equations rewritten as follows:

$$\begin{bmatrix} I & -\left(\dfrac{\partial Y_A}{\partial Y_B}\right)^* \\ -\left(\dfrac{\partial Y_B}{\partial Y_A}\right)^* & I \end{bmatrix} \begin{Bmatrix} \left(\dfrac{dY_A}{dX_A}\right)^* \\ \left(\dfrac{dY_B}{dX_A}\right)^* \end{Bmatrix} = \begin{Bmatrix} \left(\dfrac{\partial Y_A}{\partial X_A}\right)^* \\ 0 \end{Bmatrix} \tag{25}$$

The unscaled total derivatives were then recovered from the scaled values from the following expressions:

$$\frac{dY_A}{dX_A} = \frac{Y_A}{X_A}\left(\frac{dY_A}{dX_A}\right)^*$$
$$\frac{dY_B}{dX_A} = \frac{Y_B}{X_A}\left(\frac{dY_B}{dX_A}\right)^* \tag{26}$$

The response sensitivities determined from the GSE, using both direct decomposition and iterative methods, were compared with results obtained from a forward-step finite-difference approach applied to the coupled system. The percent difference in the two solutions under comparison may be written as

$$\mathrm{Per}_{i,k} = 100 \times \frac{\mathrm{ABS}\left[\left(\dfrac{dy_i}{dX_{k1}}\right) - \left(\dfrac{dY_i}{dX_{k2}}\right)\right]}{\mathrm{MAX}\left[\left(\dfrac{dy_i}{dX_{k1}}\right), \left(\dfrac{dy_i}{dX_{k1}}\right)\right]} \tag{27}$$

$$i = 1, \mathrm{NGSM}, \; k = 1, \mathrm{NX}$$

where NGSM is the dimensionality of the GSM and NX is the total number of design variables. A variance of $\mathrm{Per}_{i,k}$ is then defined as

$$\mathrm{Var}(\mathrm{Per}_{i,k}) = \frac{1}{\mathrm{NGSM}} \sum_{j=1}^{\mathrm{NGSM}} (\mathrm{Per}_{i,k})^2 \tag{28}$$

A standard deviation measure of the variance is adopted for convenience and is defined as follows:

$$\sigma(\mathrm{Per}_{i,k}) = \left[\mathrm{Var}(\mathrm{Per}_{i,k})\right]^{1/2} \tag{29}$$

Constraint Reduction

The iterative solution of the GSE was implemented in conjunction with an approach to reduce the dimensionality of the system equations. Explicitly, this involved the reduction in the total number of subsystem output parameters by an efficient constraint representation approach. Such an approach permits the representation of a large number of inequality constraints by a single cumulative measure as

$$\Omega = \left(\frac{1}{\bar{\rho}}\right) \ell n \left[\sum_{j=1}^{m} \exp(\bar{\rho} g_j)\right] \tag{30}$$

Here, Ω is the cumulative constraint measure, g_j are the m inequality constraints for the problem, and $\bar{\rho}$ is an arbitrarily large number taken to be between 25 and 50.

Solutions of the GSE were obtained for three specific cases, with selective use of the cumulative constraint to reduce the system dimensionality. A detailed definition of the dimensionality of the output vectors for each of these cases may be found in the Appendix.

Case 1

Constraint reduction techniques were not used. The system output vectors for the five mode case were as follows:

$$Y_S = (\omega^2, \phi, \bar{K}, V, I_{yy}, W, g_S) \qquad \mathrm{NS} = 453$$

$$Y_A = (L, C_{L_\alpha}, C_{M_\alpha}, g_A) \qquad \mathrm{NA} = 30$$

$$Y_C = (m_c, \delta, g_C) \qquad \mathrm{NSC} = 19 \tag{31}$$

The GSM dimensionality for this case was 502×502.

Case 2

Cumulative constraints were used for static stresses and for static and dynamic displacements, resulting in output vectors with the following dimensions for the five-mode case:

$$\mathrm{NS} = 400 \qquad \mathrm{NA} = 30 \qquad \mathrm{NSC} = 5$$

The GSM had dimensions 435×435.

Case 3

Cumulative constraints were used to represent all constraints in each subsystem, resulting in output vector dimensions as follows:

$$\mathrm{NS} = 3489 \qquad \mathrm{NA} = 25 \qquad \mathrm{NSC} = 3$$

The GSM had dimensions 417×417 for this case.

Choice of Design Variables

Since certain design variables affect the analyses and, hence, the design constraints for more than one subsystem, these design variables can be represented in more than one way in the GSM. Two representations were implemented in this study.

Case 1

In this case, design variables that contribute to more than one subsystem analysis were considered design variables in each of the subsystems. The design variable vectors were

$$X_S = (C, b, \gamma, D, a, Th) \qquad \mathrm{NXS} = 26$$

$$X_A = (C, b, \gamma, D) \qquad \mathrm{NXA} = 6$$

$$X_{SC} = (C, b, \gamma, D, G) \qquad \mathrm{NXSC} = 8,\ 12,\ 16 \tag{32}$$

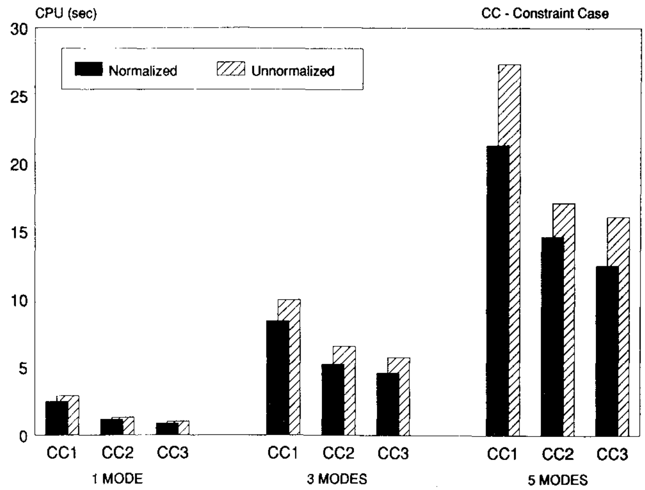

Fig. 6a Variation of computational requirements for iterative solution with system dimensionality and with constraint representation.

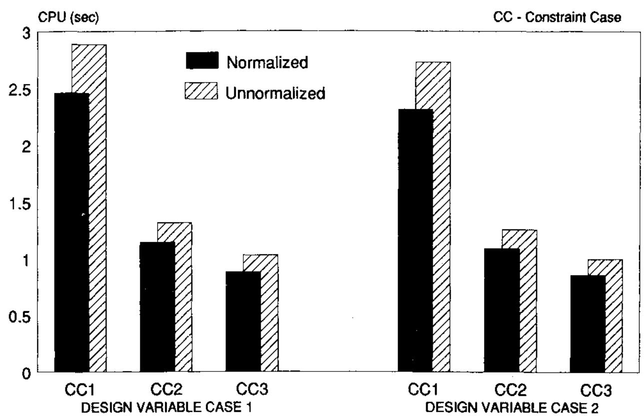

Fig. 6b Variation of computational requirements for iterative solution with design variable and constraint representation.

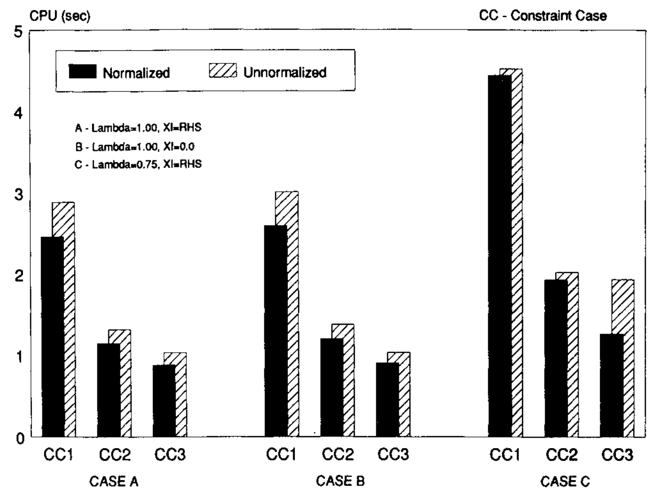

Fig. 6c Variation of computational requirements for iterative solution with different constraint representations and for variations in starting solutions and relaxation parameters.

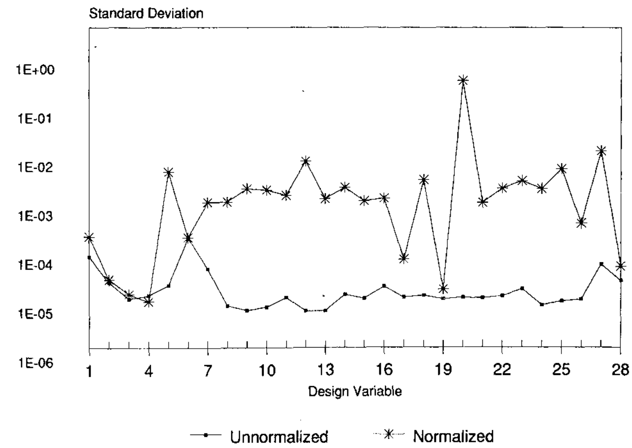

Fig. 7a Comparison of iterative and direct solutions.

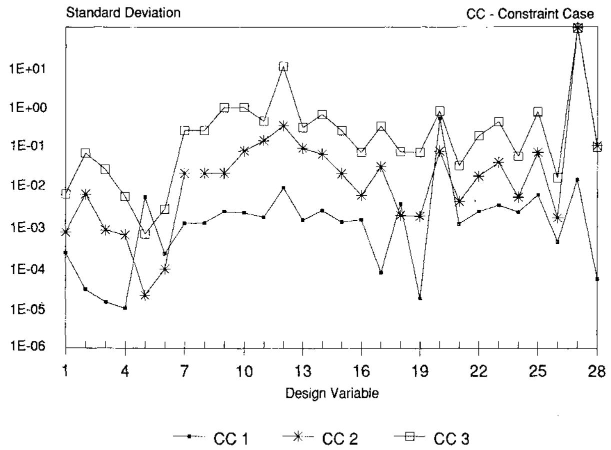

Fig. 7b Comparison of iterative and direct solutions for different constraint representations.

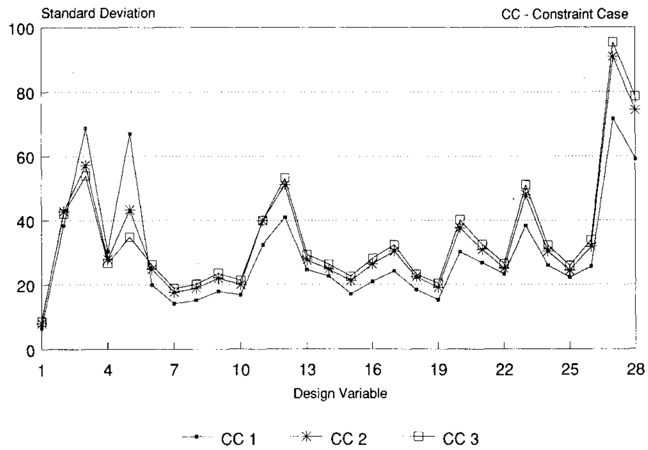

Fig. 7c Comparison of finite difference and direct solutions.

where C are chord lengths at designated span stations, b is the wing semispan, γ is the wing dihedral, D defines the wing location along the fuselage, a are the stringer areas, Th are membrane thicknesses, and G is the controller gain vector which has dimensions of two times the number of eigenmodes used in the analysis. As stated earlier, one, three, and five modes were used in the numerical work.

Case 2

In this implementation, specific design variables were allocated to only one discipline but were also represented as output

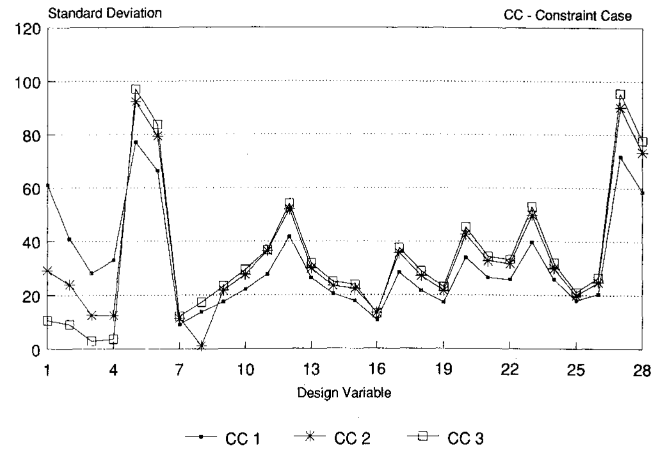

Fig. 7d Comparison of finite difference and direct solutions for different constraint representations.

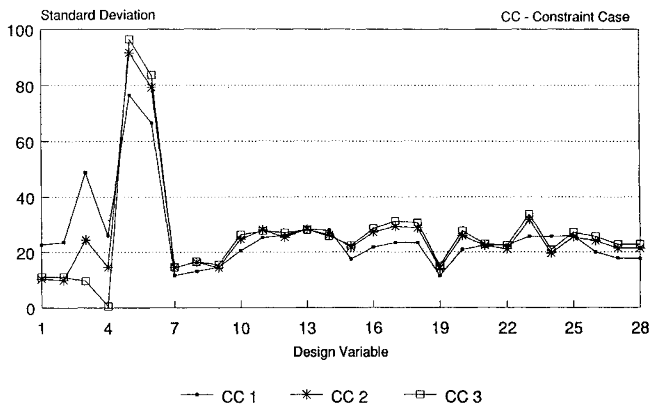

Fig. 7e Comparison of direct solutions for two different design variable representations.

in the Y vectors so their influence was still retained in other subsystem analyses. The design variable vectors were designated as follows:

$$X_S = (a, Th) \qquad NXS = 20$$

$$X_A = (C, b) \qquad NXA = 4$$

$$X_{SC} = (D, \gamma, G) \qquad NXSC = 4,\ 8,\ 12 \tag{33}$$

Here, the choice of design variables was critical, as it affected the conditioning of the GSE system matrices.

It is worthwhile to note that each of the analysis modules were independent processors, which may even reside on different computer systems. However, in the present approach, they were implemented on a single VAX 11-750 system. The flow between the independent processors was controlled in the COMMAND Language feature of DEC/VMS systems.

Discussion of Results

The effect of constraint representation, design variable allocation, and normalization of the design variables on the condition number of the GSM can be seen in Figs. 5a and 5b. As expected, condition number is seen to increase with increased dimensionality of the matrix and, even for the one-mode case, is quite large if output vector normalization is not used. The unique allocation of design variables to distinct disciplines and their inclusion in the output vector, as described in the previous section, had a minimal influence on the condition number of the GSM. As can be seen from these figures, implementation of the normalization scheme described in Eqs.

Table 1 Summary of optimization results for the aircraft problem

System	Constraint allowables	
Structures	$\omega 1_a$, Hz	2.0
	$\omega 2_a$, Hz	8.0
	d_a, in.	20.0
	$\sigma^{st}{}_a$, psi	20000.0
	$\sigma^{m}{}_a$, lb/in.	5000.0
	V_a, in.^{3}	100450.0
Aerodynamic performance	V_{st_a}, ft/s	83.6
	D_{L_a}, ft	1250.0
	D_{T0_a}, ft	1525.0
	R_a, mi	615.0
Stability/ control	dd_a, in.	10.0
	δ_a, deg	45.0
	$t_1^{1/2}{}_a$, s	45.0
	$t_2^{1/2}{}_a$, s	0.2

(23–26) significantly reduced the condition number of the GSM and contributed to an overall improvement in the solution accuracy.

The normalization scheme had the desired effect of reducing the solution times for the Gauss-Siedel iterative method, as shown in Figs. 6a and 6b. For the limited dimensionality problems considered in this work, these solution times were comparable to those required in a direct decomposition approach. However, the iterative approach would still be preferred over a direct decomposition strategy, as the round-off error accumulation in the latter can be quite severe. Figure 6c illustrates the effect of relaxation factor and the selection of an initial solution on computational requirements. As expected, if underrelaxation is used by selecting a relaxation factor less than one, the solution times show an increase. The computational requirements were minimally affected by the initial choice of the solution vector, as is seen in Fig. 6c. In an iterative framework of the design synthesis process described in Fig. 2, it is possible to use the solution of the previous iteration as the initial choice for the current iteration. The present implementation yielded mixed results with such an approach, failing to conclusively establish any distinct advantage in terms of an improvement in the overall computational requirements.

Results for the output response sensitivities were obtained by an iterative solution of the GSE, by a direct decomposition of the GSM, and by a finite-difference approximation using the coupled system equations. A comparison of these results in terms of the standard deviation measure defined in Eq. (29) for different constraint cases and design variable selections, are summarized in Figs. 7a–7e. There was extremely good agreement between the iterative and direct decomposition solutions with slightly improved agreement when design variable normalization was used. The comparison between direct decomposition solutions and finite-difference solutions also shows good agreement, with increased deviations resulting from the somewhat loose convergence criteria used in the solution of the coupled analysis equations in the finite-difference approach. Direct decomposition solutions for the two design variable representation case were also compared, and these results are presented in Fig. 7e. The present exercise did not conclusively establish the advantages of one type of design variable representation. Condition numbers for both representations were within acceptable levels and were slightly lower for case 1.

The sensitivity information obtained in the above analysis was also used in a representative optimization application. Table 1 summarizes the initial and final designs for this exercise. Results obtained for the one-mode, finite-difference solution and the three-mode, GSE solution are presented. The number of design variables for the one-mode solution are less than the three-mode solution because of the fewer gain components figuring in the design variable set. These results were obtained at the end of the sixth iteration and appear to be yielding similar values for the design objective.

Concluding Remarks

The present paper investigates an application of the GSE method in a multidisciplinary problem pertaining to integrated design synthesis. The method provides a systematic approach for decomposing a large coupled system into a series of smaller, more tractable subsystems that can be analyzed independently. The approach is, therefore, particularly well-suited for parallel computation. A hypothetical aircraft in the class of the Cessna 170-type configuration is chosen as the test environment for the investigation, with the disciplines of structures, aerodynamics, and flight mechanics contributing to the constraints and objective function for the problem. Potential drawbacks in the use of the GSE approach are identified as arising from a large number of design variables and constraints and from an improper choice of design variables. Numerical results are presented in support of the GSE approach for such large coupled design synthesis problems.

Appendix: Problem Description

Constraint Formulation

Normalized constraints of the following form

$$g = G/G_a - 1 \leq 0 \tag{A1}$$

were used in the optimization problem. Here G represents the response quantity to be constrained and subscript a denotes its prescribed allowable limit obtained from a given baseline model, thereby permitting only an improvement in the performance characteristics with optimization.

For the structures subsystem, constraints were placed on the first and second natural frequencies of the structure, on the eight lateral wingtip displacements, on the internal volume of the wing-box structure, and on the root section stresses in the stringers and membranes. Aerodynamic performance constraints included bounds on stall velocity, landing and takeoff distances over a 50-ft obstacle, and range. In this subsystem, constraints were imposed requiring ratios of midchord to root-chord and of tip-chord to midchord to be less than unity.

Allowable limits for the performance characteristics were those of a Cessna 170 aircraft. For the stability and control subsystem, constraints were placed on the dynamic lateral displacements of the wing and horizontal tail, on the deflection of the control surface, and on the times-to-half for the long- and short-period modes. Allowable limits for each subsystem are summarized in Table A1.

Design Variables

The design variable vector may be expressed in terms of a partitioned vector:

$$\{X\}^T = \{X_S | X_A | X_{SC}\} \tag{A2}$$

Six design variables that may be treated as local variables to each subsystem are as follows:

$$\{C_R, C_M, C_T, b, \gamma, D\} \tag{A3}$$

Referring to Fig. 3, C_R, C_M, C_T correspond to the chord lengths at the root, midstation, and tip of the wing. The variables b and γ correspond to the semispan and dihedral of the wing. The placement of the wing on the fuselage is determined by D, which represents the distance between the horizontal tail quarter-chord point and the trailing edge of the wing at the root section.

Structures

Additional design variables for the structures subsystem correspond to stringer and membrane thicknesses in the four prescribed wing sections of Fig. 3:

$$\{a_1, \ldots, a_8, Th_1, \ldots, Th_{12}\} \tag{A4}$$

Here, a_1 and a_2 correspond to the bottom and top stringer areas of Sec. I, a_3 and a_4 correspond to the bottom and top stringer areas of Sec. II, etc. Similarly, Th_1, Th_2, and Th_3 correspond to the bottom and top membranes and shear web thicknesses, respectively of Sec. I.

Aerodynamics

The six geometry parameters defined in Eq. (A3) were the only design variables in the aerodynamic subsystem.

Stability/Control

The gain components of the optimal control analysis were considered to be design variables in this subsystem. The gain matrix G has dimensions $n\text{ac} \times 2n\text{mod}$, where $n\text{ac}$ is the number of actuators and $2n\text{mod}$ is twice the number of modes considered in the analysis. This yielded 10 possible components of the gain matrix:

$$\{G_1, \ldots, G_{10}\} \tag{A5}$$

Constraint Reduction—Output Vector Dimensions

Dimensionality of the subsystem output vectors for the three constraint reduction representations were as follows.

Case 1

$$Y_S = \left(\omega^2_{1-5}, \phi_{1-375}, \bar{K}_{1-5}, V, I_{yy}, W, g_{s_{1-65}}\right) \qquad \text{NS} = 453$$

where $g_{S1-2} = f(\omega^2)$, $g_{S3-10} = f(d)$, $g_{S11-28} = f(\sigma^{st})$, $g_{S29-64} = f(\sigma^m)$, and $g_{S65} = f(V)$.

$$Y_A = \left(L_{1-22}, C_{L_\alpha}, C_{M_\alpha}, g_{A1-6}\right) \qquad \text{NA} = 30$$

where $g_{A1-6} = f(V_s, D_{T0}, D_L, R, C_M/C_R, C_T/C_M)$.

$$Y_{SC} = (m_c, \delta, g_{C1-17}) \qquad \text{NSC} = 19 \tag{A6}$$

where $g_{SC1-14} = f(dd)$, $g_{SC15-16} = f(t^{1/2})$, and $g_{SC17} = f(\delta)$.

Case 2

$$Y_S = \left(\omega^2_{1-5}, \phi_{1-375}, \bar{K}_{1-5}, V, I_{yy}, W, g_{s_{1-12}}\right) \qquad \text{NS} = 395$$

where $g_{S1-2} = f(\omega^2)$, $g_{S3} = f(d)$, $g_{S4-5} = f(\sigma^{st})$, $g_{S6-11} = f(\sigma^m)$, and $g_{S12} = f(V)$.

$$Y_A = \left(L_{1-22}, C_{L_\alpha}, C_{M_\alpha}, g_{A1-6}\right) \qquad \text{NA} = 30$$

where g_A has components as in case 1.

$$Y_{SC} = (m_c, \delta, g_{SC1-3}) \qquad \text{NSC} = 5 \tag{A7}$$

where $g_{SC1} = f(dd)$, $g_{SC2} = f(t^{1/2})$, and $g_{SC3} = (\delta)$.

Case 3

$$Y_S = (\omega^2_{1-5}, \phi_{1-375}, \bar{K}_{1-5}, V, I_{yy}, W, g_S) \qquad \text{NS} = 384$$

$$Y_A = \left(L_{1-22}, C_{L_\alpha}, C_{M_\alpha}, g_A\right) \qquad \text{NA} = 25$$

$$Y_{SC} = (m_c, \delta, g_{SC}) \qquad \text{NSC} = 3 \tag{A8}$$

Here, all constraints for each subsystem were expressed as a cumulative constraint.

Table A1 Allowable limits for the interacting subsystems

		Final Design	
Design variables	Initial design	One-mode, finite-difference gradients	Three-mode GSE solution
1	64.00	77.1300	71.5629
2	64.00	73.9076	68.3836
3	44.50	45.5795	62.9380
4	217.0	166.129	133.937
5	1.500	0.57240	1.75480
6	123.5	108.545	108.175
7	0.400	0.23034	0.27464
8	0.400	0.17364	0.27381
9	0.300	0.15936	0.20981
10	0.300	0.14051	0.21033
11	0.20	0.15875	0.15450
12	0.200	0.11927	0.15462
13	0.200	0.15737	0.18509
14	0.200	0.15603	0.18503
15	0.050	0.02458	0.03286
16	0.050	0.02790	0.03262
17	0.050	0.03227	0.03745
18	0.050	0.01741	0.03198
19	0.050	0.01758	0.03205
20	0.050	0.02417	0.05509
21	0.040	0.01966	0.02981
22	0.040	0.02334	0.02988
23	0.030	0.03260	0.03104
24	0.030	0.02212	0.02902
25	0.020	0.02447	0.02900
26	0.020	0.02881	0.02513
27	0.100	0.11445	0.10003
28	0.100	0.11949	0.09965
29	0.100	0.11949	0.10000
30	0.100		0.10132
31	0.100		0.07999
32	0.100		0.09999
Objective function	2033.37 lb	1909.43 lb	1888.58 lb

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